

$$\begin{aligned}
a_n &= \sum_{k=0}^{N-1} v_k e^{-jn \frac{2\pi k}{N}} \\
&= \sum_{m=0}^{N/2-1} v_{2m} e^{-jn \frac{2\pi 2m}{N}} + \sum_{m=0}^{N/2-1} v_{2m+1} e^{-jn \frac{2\pi(2m+1)}{N}} \\
&= \sum_{m=0}^{N/2-1} v_{2m} e^{-jn \frac{2\pi m}{N/2}} + e^{-jn \frac{2\pi}{N}} \sum_{m=0}^{N/2-1} v_{2m+1} e^{-jn \frac{2\pi m}{N/2}} \\
&= P_n + W_N^n Q_n
\end{aligned}$$

$$a_0 = P_0 + W_N^0 Q_0$$

$$a_1 = P_1 + W_N^1 Q_1$$

$$a_2 = P_2 + W_N^2 Q_2$$

$$\vdots$$

$$a_{N/2-1} = P_{N/2-1} + W_N^{N/2-1} Q_{N/2-1}$$

$$a_{N/2} = P_0 - W_N^0 Q_0$$

$$a_{N/2+1} = P_1 - W_N^1 Q_1$$

$$\vdots$$

$$a_{N-1} = P_{N/2-1} - W_N^{N/2-1} Q_{N/2-1}$$

$$P_{N/2+r} = P_r$$

$$Q_{N/2+r} = Q_r$$

$$W_N^{N/2+r} = -W_N^r$$

